Date: 12/10/2021

Q1. Find the ratio in which *YZ*-plane divides the line segment formed by joining the points (– 2, 4, 7) and (3, – 5, 8).

Q2. If the origin is the centroid of the triangle *PQR* with vertices *P* (2*a*, 2, 6), *Q* (– 4, 3*b*, – 10) and *R* (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Q3. Two vertices of a triangle are (1, 3, 5), (3, 2, 1) and its centroid is (– 1, 0, 0). Find the coordinates of the third vertex.

Q4. Find the ratio in which the join of the points *P* (2, – 1, 3) and *Q* (4, 3, 1) is divided by the point $R\left(\frac{20}{7}, \frac{5}{7}, \frac{15}{7}\right)$.

Q5. Find the co-ordinates of a point which devides the points (1, 3, 7), (6, 3, 2) in the ratio 2 : 3.

Q6. A point *R* with *x*-co-ordinate 4, lies on the line segment joining the points *P* (2, – 3, 4) and *Q* (8, 0, 10). Find the co-ordinates of the *R*.

Q7. The centroid of triangle *ABC* at the point *G* (1, 1, 1). If the co-ordinates of *A* and *B* are (3, – 5, 7) and (– 1, 7, – 6) respectively. Then find the co-ordinates of point *C*.

Q8. Find the co-ordinates of point *R* which devides the join of *P* (0, 0, 0) and *Q* (4, – 1, – 2) in the ratio 1 : 2, externally and verify that *P* is the mid point of *RQ*.

Q9. A point *R* with *x*-coordinate 4 lies on the line segment joining the points *P* (2, – 3, 4) and *Q* (8, 0, 10). Find the coordinates of *R*.

Q10. Find the co-ordinates of point *R* which divides *PQ* externally in the ratio 2 : 1 and verify that *Q* is the mid point of *PR*.

Q11. Find the ratio in which the line joining (2, 4, 16) and (3, 5, – 4) is devided by the plane

$$2x - 3y + z + 6 = 0$$

Q12. Find the ratio in which the line joining the points (2, 4, 5) (3, 5, – 4) is devided by *yz*-plane. And hence find its co-ordinates .

Q13. If *A* (3, 2, 0), *B* (5, 3, 2) and *C* (– 9, 6, – 3) are three points forming a triangle, *AD* is the bisector of angle *BAC*, meets *BC* at *D*. Find the co-ordinates of *D*.

Q14. Find the ratio in which *yz*-plane devides the line segment formed by joining the point (– 2, 4, 7) and (3, – 5, 8).

Q15. Find the coordinates of the point which divides the line segment joining the points (1, – 2, 3) and (3, 4, – 5) in the ratio 2 : 3; (i) internally, and (ii) externally.

Q16. Find the lengths of the medians of the triangle *A* (0, 0, 6), *B* (0, 4, 0) and *C* (6, 0, 0).

Q17. Find the ratio in which the line segment, joining the points *P* (2, 3, 4) and *Q* (– 3, 5, – 4) is divided by the *YZ*-plane. Also find the point of intersection.

Q18. Find the ratio in which the join of $A(2, 1, 5)$ and $B(3, 4, 3)$ is divided by the plane

$$2x + 2y - 2z = 1.$$

Also find the coordinates of the points of division.

Q19. Find the coordinates of the centroid of the triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

Q20. Using section formula, show that the points $A(2, -3, 4)$, $B(-1, 2, 1)$ and $C\left(0, \frac{1}{3}, 2\right)$ are collinear.

Q21. Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, – 8) is divided by YZ-plane.

Q22. Find the centroid of a triangle mid points of whose sides are (1, 2,– 3), (3, 0,1) and (– 1,1,– 4) respectively.

Q23. Show that the plane $ax + by + cz + d = 0$. Divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio

$$\frac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}$$

Q24. The midpoints of the sides of the triangle are (1, 5, – 1), (0, 4, – 2) and (2, 3, 4). Find its vertices.

Q25. Find the ratio in which the line segment joining the points (2, 4, – 3) and (– 3, 5, 4) is devided by (a) the XY-plane; (b) $x + y + z = 8$.

Q26. Find the co-ordinates of the points which divides the join of (– 2, 3, 5) and (1, – 4, – 6) in the ratio.

(a) 2 : 3 internally (b) 2 : 3 externally

Q27. Find the length of medians of the triangle $A(0, 0, 6)$, $B(0, 4, 0)$ and $C(6, 0, 0)$.

Q28. The mid-points of the sides of a triangle are (2, 2, 3), (4, 4, 2) and (3, 5, 4). Find its vertices.

Q29. Find the coordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

Q30. Find the coordinates of the foot of perpendicular drawn from the point (1, 0, 1) to the line joining the points (0, 2, 3) and (2, 1, 0).

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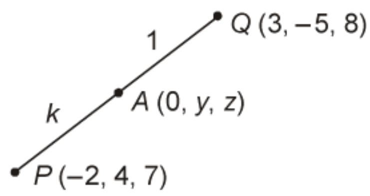
S1. Let the ratio be $k : 1$. As the point lies on YZ-plane, its x-coordinate will be 0

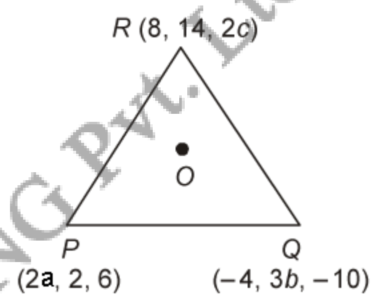


$$\Rightarrow \quad \frac{3k-2}{k+1} = 0$$

$$\Rightarrow \quad k = \frac{2}{3}$$

$\Rightarrow$ The required ratio is 2 : 3.

S2. It is given that the centroid is at the origin, we must have



$$\Rightarrow \quad \frac{2a-4+8}{3} = 0$$

$$\Rightarrow \quad \frac{2+3b+14}{3} = 0 \quad \text{and} \quad \frac{6-10+2c}{3} = 0$$

$$\Rightarrow \quad a = -2, \quad b = -\frac{16}{3}, \quad c = 2$$

S3. Let the coordinates of the third vertex be (x, y, z)

$$\Rightarrow \quad \frac{1+3+x}{3} = -1$$

$$\frac{3+2+y}{3} = 0$$

$$\frac{5+1+z}{3} = 0$$

$$\Rightarrow \quad x = -7, \quad y = -5, \quad z = -6$$

$\Rightarrow$ The required point is $(-7, -5, -6)$.

S4. Let the required ratio be $\lambda : 1$.

Then the coordinates of R are

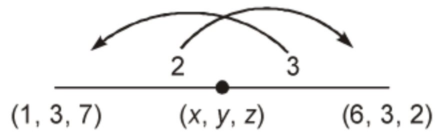
$$\left(\frac{4\lambda+2}{\lambda+1}, \frac{3\lambda-1}{\lambda+1}, \frac{\lambda+3}{\lambda+1}\right)$$

But the co-ordinates of R are $\left(\frac{20}{7}, \frac{5}{7}, \frac{15}{7}\right)$.

$\therefore \quad \dfrac{4\lambda + 2}{\lambda + 1} = \dfrac{20}{7} \quad \Rightarrow \quad \lambda = \dfrac{3}{4}$

So the required ratio is 3 : 4.

S5. Let the co-ordinates of the required point be (x, y, z)



$$\therefore \quad x = \frac{2 \times 6 + 3 \times 1}{2 + 3} = \frac{12 + 3}{5} = 3$$

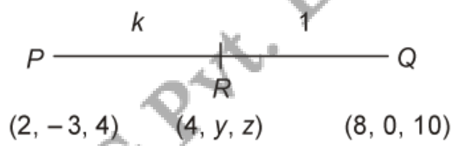
$$y = \frac{2 \times 3 + 3 \times 3}{2 + 3} = \frac{6 + 9}{5} = 3$$

$$z = \frac{2 \times 2 + 3 \times 7}{2 + 3} = \frac{4 + 21}{5} = 5$$

Required point is (3, 3, 5).

S6. Let the co-ordinates of R be $(4, y, z)$.

Let R divides PQ in ratio $k : 1$.



$$\therefore \quad R \text{ is } \left(\frac{8k + 2}{k + 1}, \frac{k \times 0 - 3}{k + 1}, \frac{10k + 4}{k + 1}\right)$$

But x-co-ordinate of R is 4.

$$\text{So,} \quad \frac{8k + 2}{k + 1} = 4 \quad \Rightarrow \quad 8k + 2 = 4k + 4$$

$$\Rightarrow \quad 4k = 2 \quad \Rightarrow \quad k = \frac{1}{2}$$

$$y = \frac{-3}{k + 1} = \frac{-3}{\frac{1}{2} + 1} = \frac{(-3) \times 2}{3} = -2$$

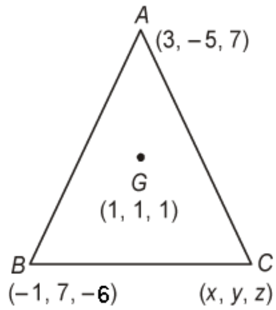
$$z = \frac{10k + 4}{k + 1} = \frac{10\left(\frac{1}{2}\right) + 4}{\frac{1}{2} + 1} = \frac{9 \times 2}{3} = 6.$$

S7. Let the co-ordinates of point C be (x, y, z)

The co-ordinate of the centroid G are

$$\left(\frac{3 - 1 + x}{3}, \frac{-5 + 7 + y}{3}, \frac{7 - 6 + z}{3}\right)$$

$$= \left(\frac{2+x}{3}, \frac{2+y}{3}, \frac{1+z}{3}\right)$$



But the co-ordinates of G are (1, 1, 1)

$\therefore \quad \frac{2+x}{3} = 1, \quad \frac{2+y}{3} = 1, \quad \frac{1+z}{3} = 1$

$\Rightarrow \quad 2 + x = 3, \quad 2 + y = 3, \quad 1 + z = 3$

$\Rightarrow \quad x = 1, \quad y = 1, \quad z = 2$

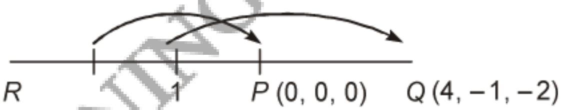
$\therefore$ The co-ordinates of C are (1, 1, 2).

S8. Let R devides the join of $P(0, 0, 0)$ and $Q(4, -1, -2)$ in the ratio 1 : 2 externally,the co-ordinates of R are

$$\left(\frac{1\times 4+(-2)\times 0}{1-2}, \frac{1\times(-1)+(-2)\times 0}{1-2}, \frac{1\times(-2)+(-2)\times 0}{1-2}\right)$$

$$= \left(\frac{4}{-1}, \frac{-1}{-1}, \frac{-2}{-1}\right) = (-4, 1, 2)$$

Now mid point of RQ is $\left(\frac{-4+4}{2}, \frac{1-1}{2}, \frac{2-2}{2}\right)$

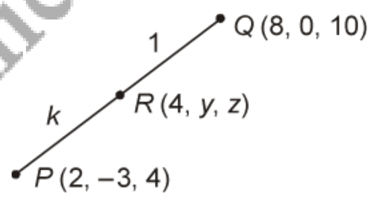


$\therefore$ Mid point of RQ is (0, 0, 0) but P is (0, 0, 0).

Thus, P is mid point of RQ.

S9. Let the coordinates of R be $(4, y, z)$

Let R divide PQ in k : 1. We must have



$$4 = \frac{8k+2}{k+1}, \quad y = \frac{0-3}{k+1}, \quad z = \frac{10k+4}{k+1}$$

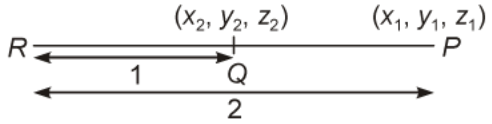
$$\Rightarrow \quad 8k + 2 = 4k + 4 \quad \Rightarrow \quad k = \frac{1}{2}$$

$$y = \frac{-3}{k+1} = \frac{-3}{\frac{1}{2}+1} = -2$$

$$z = \frac{10k+4}{k+1} = \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1} = 6$$

$\Rightarrow$ The coordinates of R will be (4, –2, 6)

S10. Let R divides the join $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in ratio 2 : 1 externally



Then the co-ordinates of R are

$$\left(\frac{2x_2 - x_1}{2-1}, \frac{2y_2 - y_1}{2-1}, \frac{2z_2 - z_1}{2-1}\right)$$

$\Rightarrow \qquad (2x_2 - x_1, 2y_2 - y_1, 2z_2 - z_1)$

Now, mid point of PR is

(x_2, y_2, z_2)

But $\quad Q$ is (x_2, y_2, z_2)

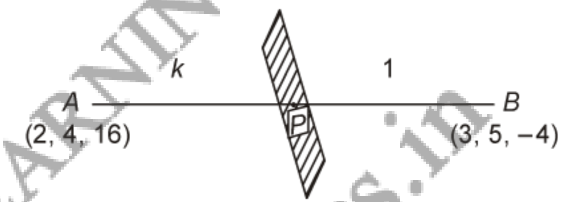
Thus, Q is the mid point of RP.

S11. Let plane $\qquad 2x - 3y + z + 6 = 0$

Devide the join of $A(2, 4, 16)$ and $B(3, 5, -4)$ at $P(x, y, z)$ in ratio $k : 1$.

Then the co-ordinates of P are

$$\left(\frac{3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{-4k+16}{k+1}\right)$$



Since P lies on the plane

$$2x - 3y + z + 6 = 0$$

$$\Rightarrow \quad 2\left(\frac{3k+2}{k+1}\right) - 3\left(\frac{5k+4}{k+1}\right) + \left(\frac{-4k+16}{k+1}\right) + 6 = 0$$

$$\Rightarrow \quad 6k + 4 - 15k - 12 - 4k + 16 + 6k + 6 = 0$$

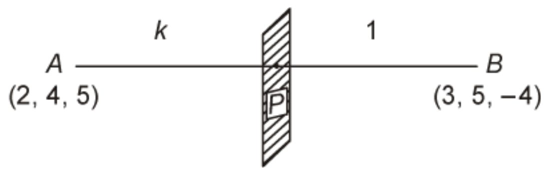
$$\Rightarrow \quad -7k + 14 = 0 \quad \Rightarrow \quad k = 2 : 1.$$

Therefore given plane devides AB in ratio 2 : 1.

S12. Let yz-plane devides the join $A(2, 4, 5)$ and $B(3, 5, -4)$ at $P(0, y, z)$ in ratio $k : 1$ where P lies on yz-plane

Then the co-ordinates of P are

$$\left(\frac{3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{-4k+5}{k+1}\right)$$



$\because \quad P$ lies on yz-plane, its x co-ordinate is zero.

$$\Rightarrow \quad \frac{3k+2}{k+1} = 0 \quad \Rightarrow \quad k = \frac{-2}{3}$$

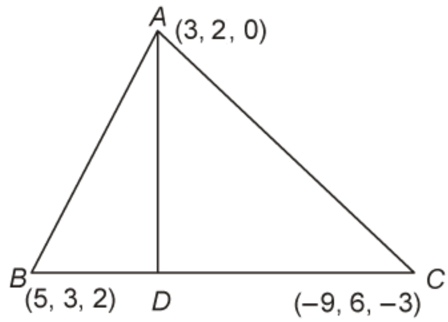
Hence ratio is $-2 : 3$.

Other co-ordinates of P

$$\left(0, \frac{-10+12}{1}, \frac{8+15}{1}\right) = (0, 2, 23)$$

Hence P is (0, 2, 23).

S13. Since AD is the bisector of angle $\angle BAC$



$$\Rightarrow \qquad \frac{BD}{DC} = \frac{AB}{AC}$$

Now, $$AB = \sqrt{(3-5)^2+(2-3)^2+(0-2)^2}$$

$$= \sqrt{4+1+4} = 3$$

$$AC = \sqrt{(3+9)^2+(2-6)^2+(0+3)^2}$$

$$= \sqrt{144+16+9} = \sqrt{169} = 13$$

$$\therefore \qquad \frac{BD}{DC} = \frac{3}{13}.$$

Hence D devides BC in the ratio 3 : 13.

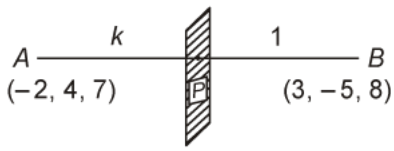
The co-ordinates of D are

$$\left(\frac{3\times(-9)+13\times 5}{3+13}, \frac{3\times 6+13\times 3}{3+13}, \frac{3\times(-3)+13\times 2}{3+13}\right) = \left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right).$$

S14. Let yz-plane devides the join of $A(-2, 4, 7)$ and $B(3, -5, 8)$ at $P(x, y, z)$ in the ratio k : 1, then the co-ordinates of P are

$$\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$$

Since P lies on yz-plane its x co-ordinate is zero.



$$\frac{3k-2}{k+1} = 0 \qquad \Rightarrow \qquad 3k-2=0$$

$$\Rightarrow \qquad k = \frac{2}{3}.$$

Therefore yz-plane divides AB in ratio 2 : 3.

S15. (i) Let $P(x, y, z)$ be the point which divides line segment joining $A(1, -2, 3)$ and $B(3, 4, -5)$ internally in the ratio 2 : 3. Therefore,

$$x = \frac{2(3)+3(1)}{2+3} = \frac{9}{5}, \qquad y = \frac{2(4)+3(-2)}{2+3} = \frac{2}{5}, \qquad z = \frac{2(-5)+3(3)}{2+3} = \frac{-1}{5}$$

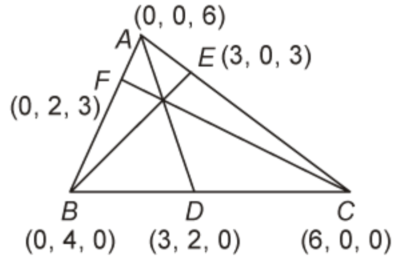
Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$.

(i) Let $P(x, y, z)$ be the point which divides line segment joining $A(1, -2, 3)$ and $B(3, 4, -5)$ externally in the ratio 2 : 3. Therefore,

$$x = \frac{2(3)+(-3)(1)}{2+(-3)} = -3, \qquad y = \frac{2(4)+(-3)(-2)}{2+(-3)} = -14, \qquad z = \frac{2(-5)+(-3)(3)}{2+(-3)} = 19$$

Therefore, the required point is $(-3, -14, 19)$.

S16. Given : $A(0, 0, 6)$, $B(0, 4, 0)$ and $C(6, 0, 0)$.



Mid-point of BC,

i.e., $$D\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) \text{ or } D(3, 2, 0)$$

Mid-point of AC,

i.e., $$E\left(\frac{0+6}{2}, \frac{0+0}{2}, \frac{6+0}{2}\right) \text{ or } (3, 0, 3)$$

Mid-point of AB,

i.e., $$F\left(\frac{0+0}{2}, \frac{0+4}{2}, \frac{6+0}{2}\right) \text{ or } F(0, 2, 3)$$

Now, $$\text{Median } AD = \sqrt{(3-0)^2+(2-0)^2+(0-6)^2}$$

$$= \sqrt{9+4+36} = 7$$

$$\text{Median } BE = \sqrt{(3-0)^2+(0-4)^2+(3-0)^2}$$

$$= \sqrt{9+16+9} = \sqrt{34}$$

and $$\text{Median } CF = \sqrt{(0-6)^2+(2-0)^2+(3-0)^2}$$

$$= \sqrt{36+4+9} = 7$$

Hence, the lengths of the medians are 7, $\sqrt{34}$ and 7.

S17. Let PQ be divided by the YZ-plane at a point R in the ratio $\lambda : 1$.

Then the coordinates of R are

$$\left(\frac{-3\lambda+2}{\lambda+1}, \frac{5\lambda+3}{\lambda+1}, \frac{-4\lambda+4}{\lambda+1}\right) \quad \text{... (i)}$$

Since R lies on YZ-plane, then x-coordinate of R is zero.

$$\therefore \quad \frac{-3\lambda+2}{\lambda+1} = 0 \quad \Rightarrow \quad \lambda = \frac{2}{3}$$

So the required ratio is 2 : 3.

Putting $\lambda = \frac{2}{3}$ in Eq. (i) the point of intersection of line segment PQ and the YZ-plane is

$$R\left(0, \frac{19}{5}, \frac{4}{5}\right).$$

S18. Suppose the given plane intersects AB at point C and let the required ratio be $\lambda : 1$. Then the coordinates of C are

$$\left(\frac{3\lambda+2}{\lambda+1}, \frac{4\lambda+1}{\lambda+1}, \frac{3\lambda+5}{\lambda+1}\right) \quad \text{... (i)}$$

Since C lies on the plane $2x + 2y - 2z = 1$. Hence

$$\therefore \quad 2\left(\frac{3\lambda+2}{\lambda+1}\right)+2\left(\frac{4\lambda+1}{\lambda+1}\right)-2\left(\frac{3\lambda+5}{\lambda+1}\right) = 1$$

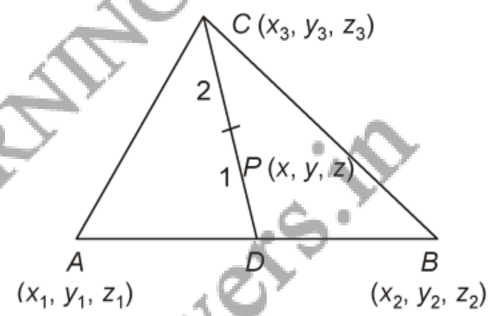
$$\Rightarrow \quad \lambda = \frac{5}{7}$$

So the required ratio is 5 : 7 and the required point of devision is putting $\lambda = \frac{5}{7}$ in Eq. (i)

$$C\left(\frac{29}{12}, \frac{9}{4}, \frac{25}{6}\right).$$

S19. D is the midpoint of AB, therefore, its coordinates are

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



The centroid divides the median in the ratio of 2 : 1. Applying the section formula, we get

$$x = \frac{2\left(\frac{x_1+x_2}{2}\right)+1(x_3)}{2+1} = \frac{x_1+x_2+x_3}{3}$$

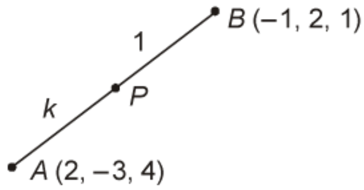
$$y = \frac{2\left(\frac{y_1+y_2}{2}\right)+1(y_3)}{2+1} = \frac{y_1+y_2+y_3}{3}$$

$$z = \frac{2\left(\frac{z_1+z_2}{2}\right)+1(z_3)}{2+1} = \frac{z_1+z_2+z_3}{3}$$

$\Rightarrow$ The centroid is

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right).$$

S20. Let the point P divide AB in the ratio $k : 1$, its coordinates will be



$$\left[\frac{-k+2}{k+1}, \frac{2k-3}{k+1}, \frac{k+4}{k+1}\right]$$

We will examine whether for some value of k the point P coincides with point C.

$$\Rightarrow \qquad \frac{-k+2}{k+1} = 0 \qquad \text{... (i)}$$

$$\frac{2k-3}{k+1} = \frac{1}{3} \qquad \text{... (ii)}$$

$$\frac{k+4}{k+1} = 2 \qquad \text{... (iii)}$$

From Eq. (i) $k = 2$, this value of k satisfies Eq. (ii) and Eq. (iii)

$\Rightarrow$ C divides AB in the ratio 2 : 1

$\Rightarrow$ A, B and C are collinear.

S21. Let YZ-plane divides the line segment joining $A(4, 8, 10)$ and $B(6, 10, -8)$ at $P(x, y, z)$ in the ratio $k : 1$. Then the coordinates of P are:

$$\left(\frac{4+6k}{k+1}, \frac{8+10k}{k+1}, \frac{10-8k}{k+1}\right)$$

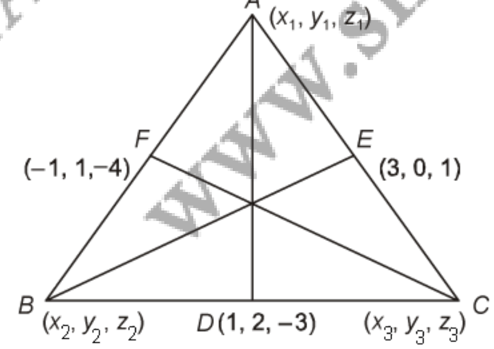
Since, P lies on the YZ-plane, its x-coordinate is zero, *i.e.*, $\frac{4+6k}{k+1} = 0$.

or $$k = -\frac{2}{3}$$

Therefore, YZ-plane divides AB externally in the ratio 2 : 3.

S22. Let the vertices of the ΔABC are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ respectively.

The mid points of BC, CA and AB are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, 4)$



Since, $\frac{x_1 + x_2}{2} = -1, \quad \frac{y_1 + y_2}{2} = 1, \quad \frac{z_1 + z_2}{2} = -4$

$\Rightarrow$ $x_1 + x_2 = -2$ ... (i)

$y_1 + y_2 = 2$ ... (ii)

$z_1 + z_2 = -8$ ... (iii)

Similarly, $\frac{x_2 + x_3}{2} = +1, \quad \frac{y_2 + y_3}{2} = 2, \quad \frac{z_2 + z_3}{2} = -3$

$\Rightarrow$ $x_2 + x_3 = 2$ ... (iv)

$y_2 + y_3 = 4$ ... (v)

$z_2 + z_3 = -6$ ... (vi)

Similarly, $x_3 + x_1 = 6$ ...(vii)

$y_3 + y_1 = 0$ ... (viii)

$z_3 + z_1 = 2$ ... (ix)

On adding Eq. (i), (iv) and (vii), we get

$$2(x_1 + x_2 + x_3) = 6 \quad \Rightarrow \quad x_1 + x_2 + x_3 = 3 \quad \text{... (x)}$$

On adding Eq. (ii), (v) and (viii), we get

$$2(y_1 + y_2 + y_3) = 6 \quad \Rightarrow \quad y_1 + y_2 + y_3 = 3 \quad \text{... (xi)}$$

On adding Eq. (iii), (vi) and (ix), we get

$$2(z_1 + z_2 + z_3) = -12 \quad \Rightarrow \quad z_1 + z_2 + z_3 = -6 \quad \text{... (xii)}$$

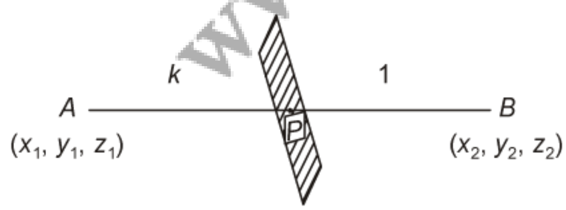
$\because$ Co-ordinates of centriod of a triangles

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$= \left(\frac{3}{3}, \frac{3}{3}, \frac{-6}{3}\right) = (1, 1, -2)$$

Hence centroid of the triangle is (1, 1, −2).

S23. Let $ax + by + cz + d = 0$ is a plane, which divides the line joining the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ at P in ratio $k : 1$.



Then the co-ordinates of P are

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}, \frac{kz_2 + z_1}{k+1}\right)$$

But P lies on the line $ax + by + cz + d = 0$. Then

$$a\left(\frac{kx_2 + x_1}{k+1}\right) + b\left(\frac{ky_2 + y_1}{k+1}\right) + c\left(\frac{kz_2 + z_1}{k+1}\right) + d = 0$$

$\Rightarrow \quad a(kx_2 + x_1) + b(ky_2 + y_1) + c(kz_2 + z_1) + d(k + 1) = 0$

$\Rightarrow \quad k(ax_2 + by_2 + cz_2 + d) + (ax_1 + by_1 + cz_1 + d) = 0$

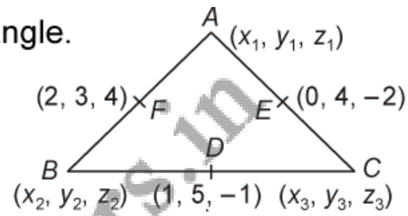
$\Rightarrow \quad k(ax_2 + by_2 + cz_2 + d) - (ax_1 + by_1 + cz_1 + d) = 0$

$\Rightarrow \quad k = \dfrac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}$

Hence ratio is

$$\frac{(ax_1 + by_1 + cz_1 + d)}{(ax_2 + by_2 + cz_2 + d)}.$$

S24. Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of the triangle.



Let $D(1, 5, -1)$, $E(0, 4, -2)$ and $F(2, 3, 4)$ be the mid points of the sides BC, CA and AB respectively. Then we have

D is the mid point of BC

$\Rightarrow \quad \dfrac{x_2 + x_3}{2} = 1, \quad \dfrac{y_2 + y_3}{2} = 5, \quad \dfrac{z_2 + z_3}{2} = -1$

$\Rightarrow \quad x_2 + x_3 = 2, \quad y_2 + y_3 = 10, \quad z_2 + z_3 = -2$ ... (i)

E is the mid point of CA

$\Rightarrow \quad \dfrac{x_1 + x_3}{2} = 0, \quad \dfrac{y_1 + y_3}{2} = 4, \quad \dfrac{z_1 + z_3}{2} = -2$

$\Rightarrow \quad x_1 + x_3 = 0, \quad y_1 + y_3 = 8, \quad z_1 + z_3 = -4$ ... (ii)

F is the mid point of AB

$\Rightarrow \quad \dfrac{x_1 + x_2}{2} = 2, \quad \dfrac{y_1 + y_2}{2} = 3, \quad \dfrac{z_1 + z_2}{2} = 4$

$\Rightarrow \quad x_1 + x_2 = 4, \quad y_1 + y_2 = 6, \quad z_1 + z_2 = 8$ ... (iii)

Adding first three Eq. in (i), (ii) and (iii), we get

$$2(x_1 + x_2 + x_3) = 2 + 0 + 4$$

$\Rightarrow \quad x_1 + x_2 + x_3 = 3$ ... (iv)

Solving first three Eq. in (i), (ii) and (iii) with Eq. (iv), we get

$$x_1 = 1, \quad x_2 = 3, \quad x_3 = -1.$$

Adding second three Eq. in (i), (ii) and (iii) we get

$$2(y_1 + y_2 + y_3) = 10 + 8 + 6$$

$$\Rightarrow \qquad y_1 + y_2 + y_3 = 12 \qquad \ldots \text{(v)}$$

Solving second three Eq. in (i), (ii) and (iii) with Eq. (v), we get

$$y_1 = 2, \quad y_2 = 4, \quad y_3 = 6.$$

Adding second three Eq. in (i), (ii) and (iii) we get

$$2(z_1 + z_2 + z_3) = -2 - 4 + 8$$

$$\Rightarrow \qquad z_1 + z_2 + z_3 = 1 \qquad \ldots \text{(vi)}$$

Solving last three Eq. in (i), (ii) and (iii) with Eq. (vi), we get

$$z_1 = 3, \quad z_2 = 5, \quad z_3 = -7.$$

Hence the vertices of triangle are $A(1, 2, 3)$, $B(3, 4, 5)$ and $C(-1, 6, -7)$.

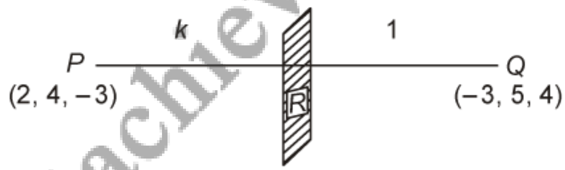
S25. Suppose the line joining the points $P(2, 4, -3)$ and $Q(-3, 5, 4)$ is devided by XY-plane at a point R in the ratio $K : 1$. Then the co-ordinates of R are

$$\left(\frac{-3k+2}{k+1}, \frac{5k+4}{k+1}, \frac{4k-3}{k+1}\right)$$

(a) Since R lies on XY-plane so its z co-ordinate must be zero. Therefore

$$\frac{4k-3}{k+1} = 0 \quad \Rightarrow \quad 4k - 3 = 0 \quad \Rightarrow \quad k = \frac{3}{4}$$

So the required ratio is $\frac{3}{4} : 1$ or $3 : 4$ internally



(b) Since the point R lies on the plane

$$x + y + z = 8$$

The co-ordinates of the point will satisfy the above equation.

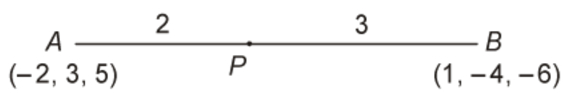
$$\Rightarrow \qquad \frac{-3k+2}{k+1} + \frac{5k+4}{k+1} + \frac{4k-3}{k+1} = 8$$

$$\Rightarrow \qquad -3k + 2 + 5k + 4 + 4k - 3 = 8k + 8$$

$$\Rightarrow \qquad -3k + 5k + 4k - 8k = 3 - 2 - 4 + 8$$

$$\Rightarrow \qquad -2k = 5 \quad \Rightarrow \quad k = \frac{-5}{2}.$$

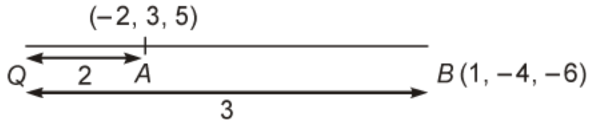
S26. (a) Let $P(x, y, z)$ divides the join of $A(-2, 3, 5)$ and $B(1, -4, -6)$ in the ratio $2 : 3$ internally. Then the co-ordinates of P are



$$\left(\frac{2\times1+3\times(-2)}{2+3}, \frac{2\times(-4)+3\times3}{2+3}, \frac{2\times(-6)+3\times5}{2+3}\right) = \left(\frac{-4}{5}, \frac{1}{5}, \frac{3}{5}\right)$$

Hence co-ordinates of P are $\left(\frac{-4}{5}, \frac{1}{5}, \frac{3}{5}\right)$.

(b) Let $Q\ (x, y, z)$ divides the join of $A\,(-2, 3, 5)$ and $B\,(1, -4, -6)$ in the ratio $2 : 3$ externally

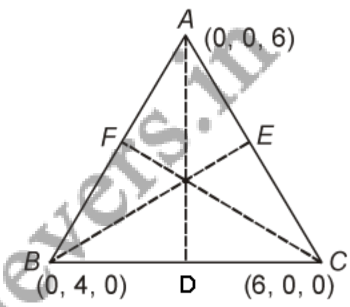


Then the co-ordinates of Q are

$$\left(\frac{2\times1-3\times(-2)}{2-3}, \frac{2\times(-4)+3\times3}{2-3}, \frac{2\times(-6)-3\times5}{2-3}\right) = (-8, 17, 27)$$

Hence co-ordinates of Q are $(-8, 17, 27)$.

S27. Here the vertices of the triangle are $A\,(0, 0, 6)$, $B\,(0, 4, 0)$ and $C\,(6, 0, 0)$.
Let D, E, F be the mid points of BC, CA and AB respectively



D is $\left(\frac{0+6}{2}, \frac{4+0}{2}, \frac{0+0}{2}\right) = (3, 2, 0)$

E is $\left(\frac{6+0}{2}, \frac{0+0}{2}, \frac{0+6}{2}\right) = (3, 0, 3)$

F is $\left(\frac{0+0}{2}, \frac{4+0}{2}, \frac{0+6}{2}\right) = (0, 2, 3)$

Median $AD = \sqrt{(3-0)^2+(2-0)^2+(0-6)^2}$

$= \sqrt{9+4+36} = 7$

Median $BE = \sqrt{(3-0)^2+(0-4)^2+(3-0)^2}$

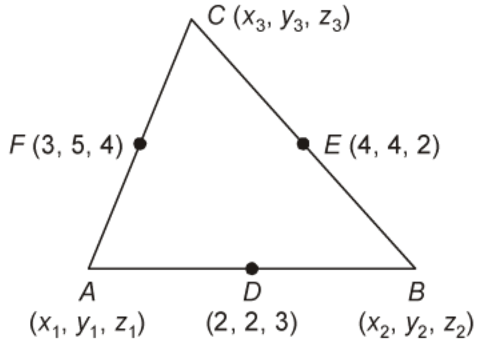
$= \sqrt{9+16+9} = \sqrt{34}$

Median $CF = \sqrt{(0-6)^2+(2-0)^2+(3-0)^2}$

$= \sqrt{36+4+9} = 7$

Length of medians are 7, $\sqrt{34}$, 7.

S28. We have,



$$\frac{x_1 + x_2}{2} = 2, \quad \frac{y_1 + y_2}{2} = 2, \quad \frac{z_1 + z_2}{2} = 3$$

$$\frac{x_2 + x_3}{2} = 4, \quad \frac{y_2 + y_3}{2} = 4, \quad \frac{z_2 + z_3}{2} = 2$$

$$\frac{x_1 + x_3}{2} = 3, \quad \frac{y_1 + y_3}{2} = 5, \quad \frac{z_1 + z_3}{2} = 4$$

$$\Rightarrow \quad x_1 + x_2 = 4, \quad y_1 + y_2 = 4, \quad z_1 + z_2 = 6$$

$$x_2 + x_3 = 8, \quad y_2 + y_3 = 8, \quad z_2 + z_3 = 4$$

$$x_1 + x_3 = 6, \quad y_1 + y_3 = 10, \quad z_1 + z_3 = 8$$

On adding we get

$$2(x_1 + x_2 + x_3) = 18, \quad 2(y_1 + y_2 + y_3) = 22, \quad 2(z_1 + z_2 + z_3) = 18$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 9, \quad y_1 + y_2 + y_3 = 11, \quad z_1 + z_2 + z_3 = 9$$

On solving, we get

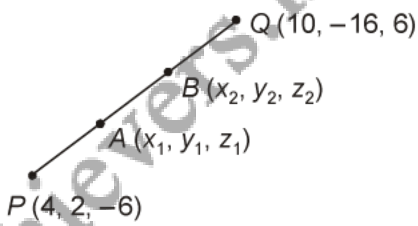
$$x_1 = 1, \quad x_2 = 3, \quad x_3 = 5$$

$$x_1 = 1, \quad x_2 = 3, \quad x_3 = 5$$

$$x_1 = 1, \quad x_2 = 3, \quad x_3 = 5$$

$\Rightarrow$ The vertices of the triangle are (1, 3, 5), (3, 1, 1) and (5, 7, 3).

S29. Let the points trisecting PQ be $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$



$\Rightarrow$ A divides PQ in 1 : 2 and B divides PQ in 2 : 1.

$$\Rightarrow \quad x_1 = \frac{10 + 8}{1 + 2} = 6$$

$$y_1 = \frac{-16 + 4}{1 + 2} = -4,$$

$$z_1 = \frac{6 - 12}{1 + 2} = -2$$

$\Rightarrow$ A will be (6, –4, –2).

Now, B divide PQ in 2 : 1.

$$x_2 = \frac{20 + 4}{2 + 1} = 8$$

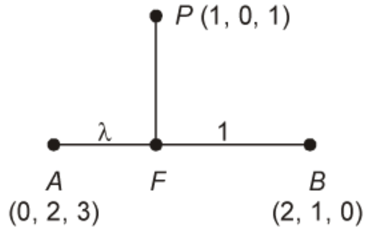
$$y_2 = \frac{-32 + 2}{2 + 1} = -10$$

$$z_2 = \frac{12-6}{2+1} = 2$$

$\Rightarrow$ B will be (8, – 10, 2).

S30. Let F, the foot of the perpendicular divide AB in the ratio of $\lambda : 1$. The coordinates of F will be,

$$\left[\frac{2\lambda}{\lambda+1}, \frac{\lambda+2}{\lambda+1}, \frac{3}{\lambda+1}\right]$$



Direction ratios of AB are

$$a_1 = 2 - 0 = 2, \quad b_1 = 1 - 2 = -1, \quad c_1 = 0 - 3 = -3$$

Direction ratios of PF,

$$a_2 = \frac{2\lambda}{\lambda+1} - 1 = \frac{\lambda-1}{\lambda+1}$$

$$b_2 = \frac{\lambda+2}{\lambda+1} - 0 = \frac{\lambda+2}{\lambda+1}$$

$$c_2 = \frac{3}{\lambda+1} - 1 = \frac{2-\lambda}{\lambda+1}$$

As $PF \perp AB$, we get

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$\Rightarrow$ $$2\left[\frac{\lambda-1}{\lambda+1}\right] - 1\left[\frac{\lambda+2}{\lambda+1}\right] - 3\left[\frac{2-\lambda}{\lambda+1}\right] = 0$$

$\Rightarrow$ $$2\lambda - 2 - \lambda - 2 - 6 + 3\lambda = 0$$

$\Rightarrow$ $$\lambda = 5/2$$

$\Rightarrow$ The coordinates of F are

$$\left[\frac{2\left(\frac{5}{2}\right)}{\frac{5}{2}+1}, \frac{\frac{5}{2}+2}{\frac{5}{2}+1}, \frac{3}{\frac{5}{2}+1}\right] = \left[\frac{10}{7}, \frac{9}{7}, \frac{6}{7}\right].$$

Date: 12/10/2021

Q1. Show that the points $P(-2, 3, 5)$, $Q(1, 2, 3)$ and $R(7, 0, -1)$ are collinear.

Q2. Prove that the points $(0, -1, -7)$ $(2, 1, -9)$ and $(6, 5, -13)$ are collinear, find the ratio in which the first two points devides the join of other two.

Q3. Using section formula prove that the three points $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

Q4. Prove that by distance formula that the points $P(1, 2, 3)$, $Q(-1, -1, -1)$ and $R(3, 5, 7)$ are collinear.

Q5. Given that $P(3, 2, -4)$ and $Q(5, 4, -6)$, $R(9, 8, -10)$ are collinear. Then find the ratio in which Q devides PR.

Q6. Using section formula prove that the three points $(-4, 6,10)$, $(2, 4,6)$ and $(14, 0,-2)$ are collinear.

Q7. If $A = (1, 2, 3)$ and $B(2, 3, 4)$ and AB is produced upto C such that $2AB = BC$ then find the co-ordinates of C.

Q8. If the points $(5, 4, 2)$, $(8, k, -7)$ and $(6, 2, -1)$ are collinear then find the value of k.

Q9. If $A = (1, 2, 3)$, $B(2, 10, 1)$, Q are collinear points and $Q_x = -1$, then Q_z = ?

Date: 12/10/2021

S1. We know that points are said to be collinear if they lie on a line.

Now,
$$PQ = \sqrt{(1+2)^2 + (2-3)^2 + (3-5)^2}$$
$$= \sqrt{9+1+4} = \sqrt{14}$$

$$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$$
$$= \sqrt{36+4+16} = \sqrt{56} = 2\sqrt{14}$$

and
$$PR = \sqrt{(7+2)^2 + (0-3)^2 + (-1-5)^2}$$
$$= \sqrt{81+9+36} = \sqrt{126} = 3\sqrt{14}$$

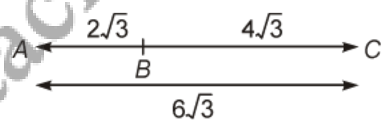
Thus, $PQ + QR = PR$.

Hence, P, Q and R are collinear.

S2. Since given points are $A(0, -1, -7)$, $B(2, 1, -9)$ and $C(6, 5, -13)$.

$$AB = \sqrt{(2-0)^2 + (1+1)^2 + (-9+7)^2}$$
$$= \sqrt{4+4+4} = \sqrt{12} = 2\sqrt{3}$$
$$BC = \sqrt{(6-2)^2 + (5-1)^2 + (-13+9)^2}$$
$$= \sqrt{16+16+16} = 4\sqrt{3}$$
$$AC = \sqrt{(6-0)^2 + (5+1)^2 + (-13+7)^2}$$
$$= \sqrt{36+36+36} = 6\sqrt{3}$$

As $(AC = BC + AB)$



$$\frac{AC}{AB} = \frac{6\sqrt{3}}{2\sqrt{3}} = \frac{3}{1}$$

$\therefore$ $AC : AB = 3 : 1$. Externally.

S3. Let the points be $A(-2, 3, 5)$, $B(1, 2, 3)$ and $C(7, 0, -1)$ are collinear.

Let B devides the join of A and C in ratio

$$\Rightarrow \quad \left(\frac{-2+7k}{k+1}, \frac{3}{k+1}, \frac{5-k}{k+1}\right) = (1, 2, 3)$$

$$\Rightarrow \quad \frac{-2+7k}{k+1} = 1 \qquad \text{... (i)}$$

$\Rightarrow$ $$\frac{3}{k+1} = 2 \quad \text{... (ii)}$$

$\Rightarrow$ $$\frac{5-k}{k+1} = 3 \quad \text{... (iii)}$$

After solving each Eq., we get

$$k = \frac{1}{2}$$

Therefore, B devides the join of A and C in the ratio 1 : 2.

Hence, A, B, C are collinear.

S4. We have,

$$PQ = \sqrt{(-1-1)^2 + (-1-2)^2 + (-1-3)^2}$$

$$= \sqrt{4+9+16} = \sqrt{29}$$

$$QR = \sqrt{(3+1)^2 + (5+1)^2 + (7+1)^2}$$

$$= \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$$

and $$PR = \sqrt{(3-1)^2 + (5-2)^2 + (7-3)^2}$$

$$= \sqrt{4+9+16} = \sqrt{29}$$

SInce $QR = PQ + PR$, therefore the points are collinear.

S5. Let $Q(5, 4, -6)$ devides join of $P(3, 2, -4)$ and $R(9, 8, -10)$ in ratio $K : 1$.

Then the co-ordinates of Q are

$$\left(\frac{9k+3}{k+1}, \frac{8k+2}{k+1}, \frac{-10k-4}{k+1}\right)$$

But the co-ordinate of Q is $(5, 4, -6)$ hence

$$\frac{9k+3}{k+1} = 5 \quad \text{... (i)}$$

$$\frac{8k+2}{k+1} = 4 \quad \text{.... (ii)}$$

$$\frac{-10k-4}{k+1} = -6 \quad \text{... (iii)}$$

From each of the Eq. we get. $k = \frac{1}{2}$ hence Q devides PR in ratio 1 : 2 internally.

S6. If points (–4, 6, 10), (2, 4, 6) and (14, 0, 2) are collinear.

Let point (2, 4, 6) devides the join of (–4, 6, 10) and (14, 0, –2) in the ratio k : 1.

$$\Rightarrow \quad \left(\frac{14k-4}{k+1}, \frac{6}{k+1}, \frac{-2k+10}{k+1}\right) = (2, 4, 6)$$

$$\Rightarrow \quad \frac{14k-4}{k+1} = 2 \qquad \text{... (i)}$$

$$\Rightarrow \quad \frac{6}{k+1} = 4 \qquad \text{... (ii)}$$

$$\Rightarrow \quad \frac{-2k+10}{k+1} = 6 \qquad \text{... (iii)}$$

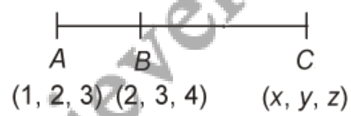
After solving each Eq., we get

$$k = \frac{1}{2}$$

Hence point (2, 4, 6) devides the join of points (–4, 6, 10) and (14, 0, –2) in the ratio $\frac{1}{2} : 1 \Rightarrow 1 : 2$

Hence the given points are collinear.

S7. $\because \quad 2AB = BC$



$$\Rightarrow \quad \frac{AB}{BC} = 1 : 2$$

Let the co-ordinates of point C be (x, y, z)

$$\therefore \quad 2 = \frac{1 \times 2 + x \times 1}{1+2} \quad \Rightarrow \quad 2 + x = 6$$

$$\Rightarrow \quad x = 4.$$

$$3 = \frac{2 \times 2 + y \times 1}{1+2} \quad \Rightarrow \quad 4 + y = 9$$

$$\Rightarrow \quad y = 5$$

$$4 = \frac{3 \times 2 + z \times 1}{1+2} \quad \Rightarrow \quad 6 + z = 12$$

$$\Rightarrow \quad z = 6$$

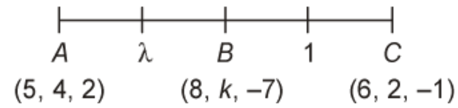
hence point C is (4, 5, 6).

S8. Let $A = (5, 4, 2),\quad B = (8, k, -7)$ and $C(6, 2, -1)$

Let B devides AC in ratio $\lambda : 1$

$$\therefore \qquad 8 = \frac{6\lambda + 5}{\lambda + 1}$$

$$\Rightarrow \qquad 8\lambda + 8 = 6\lambda + 5 \qquad \Rightarrow \qquad 2\lambda = -3$$



$$\Rightarrow \qquad \lambda = \frac{-3}{2}$$

$$k = \frac{2\lambda + 4}{\lambda + 1} \qquad \Rightarrow \qquad k = \frac{2 \times \left(\frac{-3}{2}\right) + 4}{\frac{-3}{2} + 1}$$

$$\Rightarrow \qquad k = \frac{-3 + 4}{\frac{-1}{2}} = \frac{1}{\left(\frac{-1}{2}\right)} = -2$$

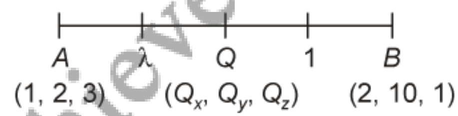
Hence $k = -2$.

S9. Let co-ordinates of Q are (Q_x, Q_y, Q_z) respectively.

Let Q devides AB in ratio $\lambda : 1$

$$\because \qquad Q_x = -1 \qquad \text{(Given)}$$

$$-1 = \frac{1 \times 1 + 2 \times \lambda}{\lambda + 1} \qquad \Rightarrow \qquad 2\lambda + 1 = -\lambda - 1$$



$$\Rightarrow \qquad 3\lambda = -2 \qquad \Rightarrow \qquad \lambda = \frac{-2}{3}$$

$$Q_z = \frac{1 \times \lambda + 3 \times 1}{\lambda + 1}$$

$$Q_z = \frac{\lambda + 3}{\lambda + 1} = \frac{\frac{-2}{3} + 3}{\frac{-2}{3} + 1}$$

$$= \frac{\frac{-2 + 9}{3}}{\frac{-2 + 3}{3}} = +7$$

Hence $Q_z = +7$.